

# Physics 2015 (Outside Delhi)

# SET I

Time allowed : 3 hours

Maximum marks : 70

## SECTION-A

1. Define the term 'self-inductance' of a coil. Write its S.I. unit. [1]

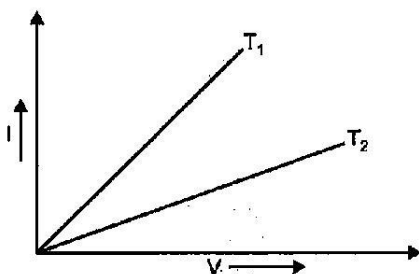
**Answer :** Self-inductance of a coil is numerically equal to the amount of magnetic flux linked with the coil when unit current flows through the coil. The S.I. unit of self inductance is henry (H) or weber per ampere.

$$1 \text{ H} = 1 \text{ Wb/A}$$

2. Why does bluish colour predominate in a clear sky? [1]

**Answer :** While light from the sun reaches the atmosphere that is comprised of the tiny particles of the atmosphere. These act as a prism and cause the different components to scatter. As blue light travels in shorter and smaller waves in comparison to the other colours of the spectrum, it is scattered the most, causing the sky to appear bluish.

3. I-V graph for a metallic wire at two different temperatures,  $T_1$  and  $T_2$  is as shown in the figure. Which of the two temperature is lower and why? [1]



\*\*Answers is not given due to change in the present syllabus.

**Answer :** The slope of a V-I graph is given by

the formula  $\frac{I}{V} = \frac{1}{R}$ . Thus, the smaller the slope larger is the resistance. As the resistance of a metal increases with the increase in temperature, so resistance at  $T_2$  is higher and  $T_1$  is lower.

4. Which basic mode of communication is used for telephonic communication? \*\* [1]

5. Why do the electrostatic field lines not form closed loops? [1]

**Answer :** Electrostatic field lines never form loops because they do not converge at only one point as in the case with magnetic field lines. They depend on +ve and -ve charges that can extend to infinity in any particular direction.

## SECTION-B

6. When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de-Broglie wavelength associated with the electron change? Justify your answer. [2]

**Answer :** de-Broglie wavelength associated with a moving charge particle having a K.E. 'K' can be given as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \left[ K = \frac{1}{2} mv^2 = \frac{p^2}{2m} \right] \dots (i)$$

The kinetic energy of the electron in any orbit of hydrogen atom can be given as

$$K = -E = -\left( \frac{13.6}{n^2} \text{ eV} \right) = \frac{13.6}{n^2} \text{ eV} \dots (ii)$$

Let  $K_1$  and  $K_4$  be the K.E. of the electron in ground state and third excited state, where  $n_1 = 1$  shows ground state and  $n_2 = 4$  shows third excited state. Using the concept of equations (i) and (ii), we have

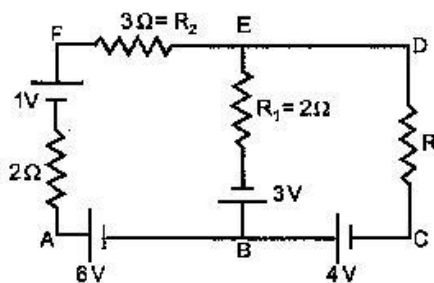
$$\frac{\lambda_1}{\lambda_4} = \sqrt{\frac{K_4}{K_1}} = \sqrt{\frac{n_1^2}{n_2^2}}$$

$$\frac{\lambda_1}{\lambda_4} = \sqrt{\frac{1^2}{4^2}} = \frac{1}{4}$$

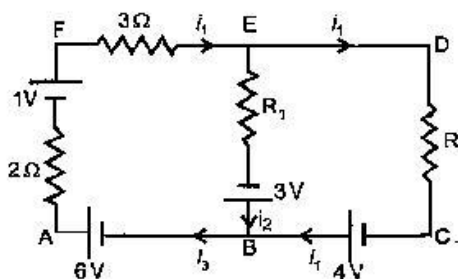
$$\Rightarrow \lambda_1 = \frac{\lambda_4}{4}$$

i.e., the wavelength in the ground state will decrease.

7. Write two factors which justify the need of modulating a low frequency signal into high frequencies before transmission.\*\* [2]
8. Use Kirchhoff's rules to determine the potential difference between the points A and D when no current flows in the arm BE of the electric network shown in the figure. [2]



Answer:



According Kirchhoff's junction law at B,

$$i_3 = i_1 + i_2$$

As  $i_2 = 0$  (given)

$$\therefore i_3 = i_1$$

Applying kirchhoff's second law to loop AFEB,

$$\therefore i_3 \times 2 + i_1 \times 3 + i_2 R_1 = 1 + 3 + 6$$

$$2i_3 + 3i_3 = 10$$

$$5i_3 = 10$$

$$i_3 = i_1 = 2 \text{ A}$$

The potential difference between A to D along the branch AFD,

$$\begin{aligned} \therefore V_{AD} &= 2i_3 - 1 + 3 \times i_1 \\ &= (4 - 1 + 6) \text{ V} \\ &= 9 \text{ V} \end{aligned}$$

9. You are given two converging lenses of focal lengths 1.25 cm and 5 cm to design a compound microscope. If it is desired to have a magnification of 30, find out the separation between the objective and the eyepiece. [2]

OR

A small telescope has an objective lens of focal length 150 cm and eye-piece of focal length 5 cm. What is the magnifying power of the telescope for viewing distant objects in normal adjustment ?

If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens ?

Answer: Given,  $f_e = 5 \text{ cm}$ ;  $f_o = 1.25 \text{ cm}$  and  $M = -30$ .

Let L be the tube length (distance between the objective and the eyepiece)

$$\therefore M = \frac{-L}{f_o} \left( 1 + \frac{d}{f_e} \right)$$

[d is a constant and equals to the normal distance of clear vision of the human eye]

$$\text{Hence, } -30 = -\frac{L}{1.25} \left( 1 + \frac{25}{5} \right)$$

$$\begin{aligned} \therefore L &= \frac{30 \times 1.25}{6} \\ &= 6.25 \text{ cm} \end{aligned}$$

Hence, the tube length = 6.25 cm.

OR

If the telescope is in normal adjustment, i.e., the final image is at infinity.

$$M = \frac{f_o}{f_e}$$

Since  $f_o = 150 \text{ cm}$ ,  $f_e = 5 \text{ cm}$

$$M = \frac{150}{5} = 30$$

If tall tower is at distance 3 km from the objective lens of focal length 150 cm. It will form its image at distance  $v_o$ . So,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{150 \text{ cm}} = \frac{1}{v_o} - \frac{1}{(-3 \text{ km})}$$

$$\frac{1}{v_o} = \frac{1}{1.5 \text{ m}} + \frac{1}{3000 \text{ m}}$$

$$v_o = \frac{3000 \times 1.5}{3000 - 1.5} = \frac{4500}{2998.5} = 1.5 \text{ m}$$

$$\text{Magnification, } m_o = \frac{I}{O} = \frac{h_i}{h_o} = \frac{v_o}{u_o}$$

$$\frac{h_i}{100 \text{ m}} = \frac{1.5 \text{ m}}{3 \text{ km}} = \frac{1.5 \text{ m}}{3000 \text{ m}}$$

$$h_i = \frac{1.5 \times 100}{3000} = \frac{1}{20} \text{ m}$$

$$h_i = 0.05 \text{ m}$$

10. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared, visible, ultraviolet) of hydrogen spectrum does this wavelength lie? [2]

**Answer :** The formula for wavelength ( $\lambda$ ) in Balmer series is :

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \text{ where } n = 3, 4, 5 \text{ and}$$

$R = 1.097 \times 10^7 \text{ m}^{-1}$  is the Rydberg constant.

Now, the shortest wavelength in Balmer series when  $n = \infty$  is

$$\frac{1}{\lambda} = 1.097 \times 10^7 \text{ m}^{-1} \times \left( \frac{1}{2^2} - \frac{1}{\infty^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times (0.25 - 0)$$

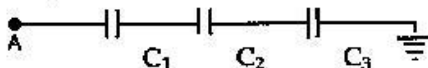
$$\frac{1}{\lambda} = 0.2743 \times 10^7$$

$$\begin{aligned} \lambda &= 3.6456 \times 10^{-7} \\ &= 364.56 \times 10^{-9} \text{ m} \\ &= 364.56 \text{ nm.} \end{aligned}$$

This wavelength lies in the visible region of the hydrogen spectrum.

### SECTION - C

11. Calculate the potential difference and the energy stored in the capacitor  $C_2$  in the circuit shown in the figure. Given potential at A is 90 V,  $C_1 = 20 \mu\text{F}$ ,  $C_2 = 30 \mu\text{F}$  and  $C_3 = 15 \mu\text{F}$ . [3]



**Answer :** The given capacitors are connected in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$

$$= \frac{3+2+4}{60} = \frac{9}{60} = \frac{3}{20}$$

$$C = \frac{20}{3} \mu\text{F},$$

As the capacitors are connected in series, so the charge remains same.

$$\therefore Q = CV = \frac{20}{3} \times 90 = 600 \mu\text{C}$$

Potential difference across  $C_2$ ,

$$\begin{aligned} V_2 &= \frac{Q}{C_2} \\ &= \frac{600}{30} = 20 \text{ V} \end{aligned}$$

Energy stored in capacitor  $C_2$ ,

$$\begin{aligned} U &= \frac{1}{2} C_2 (V_2)^2 \\ &= \frac{1}{2} \times 30 \times 10^{-6} \times (20)^2 \\ &= \frac{1}{2} \times 30 \times 10^{-6} \times 400 \\ &= 6 \times 10^{-3} \text{ J} \end{aligned}$$

12. Find the relation between drift velocity and relaxation time of charge carriers in a conductor. A conductor of length  $L$  is connected to a d.c. source of emf 'E'. If the length of the conductor is tripled by stretching it, keeping 'E' constant, explain how its drift velocity would be affected. [3]

**Answer :** All free electrons suffer collisions with the heavy fixed ions inside the conductor. After collisions, these electrons again emerge with the same speed, but in random directions. So, at given time, net velocity of the electrons is zero i.e.,

$$u_{\text{avg}} = \frac{u_1 + u_2 + \dots + u_n}{n} = 0$$

If the electric field established inside the conductor, electrons get accelerated, so

$$a = -\frac{eE}{m}$$

Now, the average velocity of all electrons is given by

$$v_d = u_{\text{avg}} - \frac{eE}{m} \tau$$

$$\vec{v}_d = 0 - \frac{eE}{m} \tau$$

where  $\tau$  is the average relaxation time.

But 
$$E = \frac{V}{l}$$

$$\vec{v}_d = - \frac{eV\tau}{ml}$$

$$v_d \propto \frac{1}{l}$$

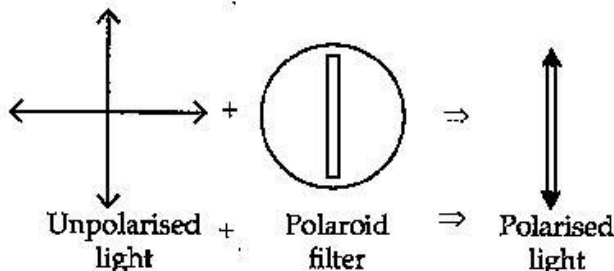
As the drift velocity is inversely proportional to length of conductor, so the drift velocity would be reduced by one third if the length of the conductor is tripled.

13. State clearly how an unpolarised light gets linearly polarised when passed through a polaroid. [3]

(i) Unpolarised light of intensity  $I_0$  is incident on a polaroid  $P_1$  which is kept near another polaroid  $P_2$  whose pass axis is parallel to that of  $P_1$ . How will the intensities of light,  $I_1$  and  $I_2$ , transmitted by the polaroids  $P_1$  and  $P_2$  respectively, change on rotating  $P_1$  without disturbing  $P_2$ ?

(ii) Write the relation between the intensities  $I_1$  and  $I_2$ .

**Answer :** Polaroid filters are made of a special material that is capable of blocking of the two planes of vibration of an electromagnetic wave. In this sense, a polaroid acts as a device that filter out half of the vibrations on transmission of the light through the filter. When unpolarised light is transmitted through a polaroid filter, it emerge with one-half the intensity and with vibrations in a single plane; it emerges as polarised light.



(i) When an unpolarised light of intensity  $I_0$  is incident on a polaroid  $P_1$  of the light intensity  $I_1$  will become half of the incident intensity.

$$I_1 = \frac{I_0}{2}$$

and when it will pass through  $P_2$  polaroid kept near  $P_1$  whose pass axis is parallel to that of  $P_1$ ,

then, there is no change in the intensity but the angle will change by  $\cos^2 \theta$

$$I_2 = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta$$

(ii) The relation between  $I_1$  and  $I_2$  is given by

$$\frac{I_2}{I_1} = \cos^2 \theta$$

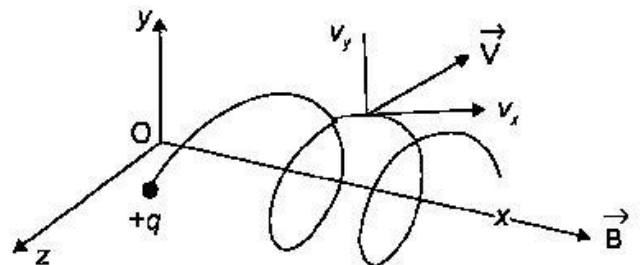
14. Define modulation index. Why is its value kept, in practice, less than one ?

A carrier wave of frequency 1.5 MHz and amplitude 50 V is modulated by a sinusoidal wave of frequency 10 kHz producing 50% amplitude modulation. Calculate the amplitude of the AM wave and frequencies of the side bands produced.\*\* [3]

15. A uniform magnetic field  $B$  is set up along the positive X-axis. A particle of charge 'q' and mass 'm' moving with a velocity  $v$  enters the field at the origin in X-Y plane such that it has velocity components both along and perpendicular to the magnetic field  $B$ . Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation. [3]

**Answer :** If component  $v_x$  of the velocity vector is along the magnetic field, and remain constant, the charge particle will follow a helical trajectory; as shown in figur.

If the velocity component  $v_y$  is perpendicular to the magnetic field  $B$ , the magnetic force acts like a centripetal force  $qv_y B$ .



So 
$$qv_y B = \frac{mv_y^2}{r}$$

$$v_y = \frac{qBr}{m}$$

Since tangent velocity  $v_y = r\omega$

$$\rightarrow r\omega = \frac{qBr}{m}$$

$$\Rightarrow \omega = \frac{qB}{m}$$

Time taken for one revolution,

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

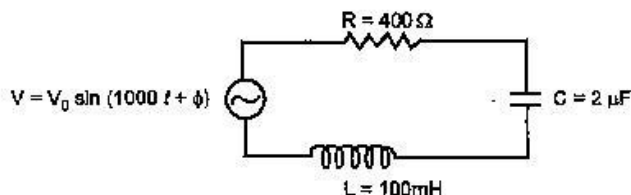
[ $\because W = 2\pi r$ ]

and the distance moved along the magnetic field in the helical path is

$$x = v_x T$$

$$= v_x \frac{2\pi m}{qB}$$

16. (a) Determine the value of phase difference between the current and the voltage in the given series LCR circuit



- (b) Calculate the value of the additional capacitor which may be joined suitably to the capacitor C that would make the power factor of the circuit unity. [3]

Answer : (a)  $V = V_0 \sin(1000t + \phi)$

Comparing this with  $V = V_0 \sin(\omega t + \phi)$

we have,

$$\omega = 1000$$

Given :  $L = 100 \times 10^{-3} \text{ H}$ ,  $R = 400 \Omega$ ,  $C = 2 \mu\text{F}$

$$X_L = \omega L$$

$$X_L = 1000 \times 100 \times 10^{-3} = 100 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 2 \times 10^{-6}} =$$

$$= \frac{1}{2} \times 10^3$$

$$= 500 \Omega$$

Phase difference,

$$\tan \phi = \frac{X_L - X_C}{R}$$

$$\tan \phi = \frac{X_C - X_L}{R}, X_C > X_L$$

we have,

$$R = 400 \Omega$$

$$\tan \phi = \frac{500 - 100}{400} = \frac{400}{400} = 1$$

$$\tan \phi = 1$$

$\therefore \tan \phi = \tan 45^\circ$

$$\phi = 45^\circ$$

- (b) The power factor of the circuit is unity. It means that the given circuit is in resonance. It is possible, if another capacitor  $C'$  is used in

the circuit.

$$X_{C'} = X_L$$

$$\frac{1}{\omega C'} = \omega L$$

$$C' = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2 \times 100 \times 10^{-3}}$$

$$C' = 10 \mu\text{F}$$

Since  $C' > C$ , so an additional capacitor of  $8 \mu\text{F}$  would be connected in parallel to the capacitor of  $C = 2 \mu\text{F}$ .

17. Write the expression for the generalized form of Ampere's circuital law. Discuss its significance and describe briefly how the concept of displacement current is explained through charging/discharging of a capacitor in an electric circuit. [3]

Answer : Generalized form of Ampere circuital law :

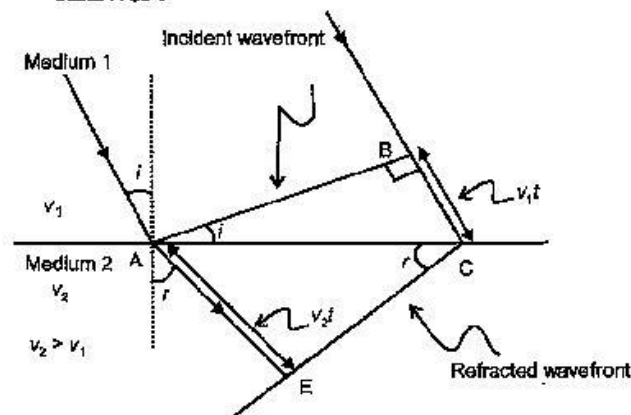
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left( I_c + \epsilon_0 \frac{d\phi}{dt} \right)$$

It signifies that the source of magnetic field is not just due to the conduction electric current due to flow of charge but also due to the time rate of change of electric field called displacement current.

During charging and discharging of a capacitor the electric field between the plates will change, because of which there will be a change of electric flux (displacement current) between the plates.

18. Use Huygen's principle to show how a plane wavelength propagates from a denser to rarer medium. Hence verify Snell's law of refraction. [3]

Answer :



In  $\Delta ABC$ ,

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC} \quad (i)$$

In  $\Delta CEA$ ,

$$\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC} \quad (ii)$$

From equation (i) and (ii)

$$\therefore \frac{\sin i}{\sin r} = \frac{BC}{AE} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} \quad \text{(iii)}$$

$$\therefore \text{But} \quad \mu_1 = \frac{c}{v_1}$$

$$\mu_2 = \frac{c}{v_2}$$

$$\therefore \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} \quad \text{(iv)}$$

on comparing equation (iii) and (iv)

$$\therefore \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\text{or} \quad \mu_2 \sin r = \mu_1 \sin i.$$

It is Snell's law.

19. Identify the gates P and Q shown in the figure. Write the truth table for the combination of the gates shown.



Name the equivalent gate representing this circuit and write its logic symbol.\*\* [3]

20. Draw a circuit diagram of a C.E. transistor amplifier. Briefly explain its working and write the expression for (i) current gain, (ii) voltage gain of the amplifier.\*\* [3]

21. (a) Write three characteristic properties of nuclear force.

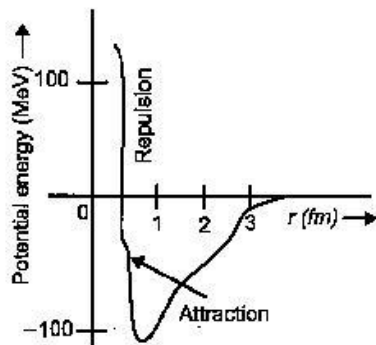
(b) Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions that can be drawn from the graph. [3]

Answer : (a) 1. Nuclear forces are short range forces ;

2. Nuclear forces are primarily attractive and extremely strong;

3. Nuclear forces are charge independent.

(b)



22. (a) Describe briefly three experimentally observed features in the phenomenon of photoelectric effect.

(b) Discuss briefly how wave theory of light cannot explain these features. [3]

OR

(a) Write the important properties of photons which are used to establish Einstein's photoelectric equation.

(b) Use this equation to explain the concept of (i) threshold frequency and (ii) stopping potential.

Answer : (a) 1. The photoelectric effect will not occur when the frequency of the incident light is less than the threshold frequency. Different materials have different threshold frequencies and most elements have threshold frequencies in the ultraviolet region of the electromagnetic spectrum.

2. The maximum KE of a stream of photoelectrons increases linearly with the frequency of the incident light above the threshold frequency.

3. The rate at which photoelectrons are emitted from a photosensitive surface is directly proportional to the intensity of incident light when the frequency is constant.

(b) Classical wave theory cannot explain :

1. The existence of threshold frequency because it predicts that electrons would absorb enough energy to escape and there would not be any threshold frequency.

2. The almost immediate emission of photoelectrons as, according to this theory, electrons require a period of time before sufficient energy is absorbed by it to escape from the metal; however, such a thing does not happen practically.

3. It cannot explain why maximum KE is dependent on the frequency and independent of intensity.

OR

(a) The important property of photons that is useful in establishing Einstein's photoelectric equation is their ability to hold on to the electrons of an atom by their forces of attraction.

(b) Einstein's photoelectric equation states that :

$$E_{\max} = h\nu - \phi$$

$$\text{If } E_{\max} > 0 \text{ then } h\nu - \phi > 0$$

$$\text{or} \quad h\nu - h\nu_0 > 0$$

Here,  $\nu_0$  is the threshold frequency.

Again, loss in KE = gain in electromagnetic PE.

$$\text{or} \quad E_{\max} = eV_s$$

where  $V_s$  represents the stopping potential.

## SECTION-D

23. One morning an old man walked bare-foot to replace the fuse wire in kit kat fitted with power supply mains for his house. Suddenly he screamed and collapsed on the floor. His wife cried loudly for help. His neighbour's son Anil heard the cries and rushed to the place with shoes on. He took a wooden baton and used it to switch off the main supply. [4]

Answer the following questions :

- (i) What is the voltage and frequency of mains supply in India ?
- (ii) These days most of the electrical devices we use require a.c. voltage. Why ?
- (iii) Can a transformer be used to step up d.c. voltage ?
- (iv) Write two qualities displayed by Anil by his action.\*\*

Answer : (i) The voltage and frequency of mains supply in India are 240 V and frequency is 50 Hz.

(ii) Most electrical devices require a.c. voltage because a.c. is available by default through the mains supply and also because d.c., is actually a one-way current and is available only through batteries.

(iii) No, a transformer cannot be used to step-up d.c. voltage because induced emf in the secondary coil of the transformer is only due to change of magnetic flux in primary coil.

## SECTION - E

24. (a) Define electric flux. Write its S.I. unit.

"Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is". Justify this statement with the help of a suitable example.

- (b) Use Gauss's law to prove that electric field inside a uniformly charged spherical shell is zero. [5]

OR

- (a) Derive the expression for the energy stored in a parallel plate capacitor. Hence obtain the expression for the energy density of the electric field.
- (b) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor.

Answer : (a) The electric flux through an area is defined as the electric field multiplied by the area of the surface projected on a plane, perpendicular to the field. Its S.I. unit is voltmeter (Vm) or Newton metre square per coulomb (Nm<sup>2</sup> C<sup>-1</sup>).

The given statement is justified because while measuring the flux, the surface area is more important than its volume or its size.

(b) To prove that the electric field inside a uniformly charged spherical shell is zero, we place a single positive point charge 'q' at the centre of an imaginary spherical surface with radius R. The field lines of this point radiate outside equally in all directions. The magnitude E of the electric field at every point on the surface

$$\text{is given by } E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{R^2}.$$

At each point on the surface,  $\vec{E}$  is  $\perp$  to the surface and its magnitude is the same.

Thus, the total electric flux ( $\phi_E$ ) is the product of their field magnitude E and the A.

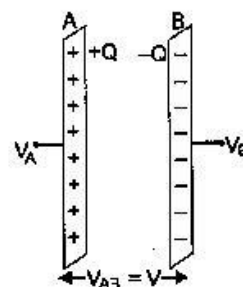
$$\begin{aligned} \text{Hence, } \phi_E &= EA \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} (4\pi\epsilon R^2) \\ &= \frac{q}{\epsilon_0} \end{aligned}$$

If the sphere is uniformly charged, then there is zero charge inside the sphere, according to Gauss's law

$$\text{When } q = 0, \phi_E = \frac{0}{\epsilon_0} = 0$$

OR

- (a) Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Initial potential difference between capacitor plates is zero. Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the



potential difference between its plates  $V = \frac{q}{C}$ .

Now work done in giving an additional infinitesimal charge  $dq$  to capacitor

$$dW = V dq = \frac{q}{C} dq$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration. Therefore, total work done.

$$W = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq$$

$$= \frac{1}{C} \left[ \frac{q^2}{2} \right]_0^Q = \frac{1}{C} \left( \frac{Q^2}{2} - \frac{0}{2} \right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then  $Q = CV$

$$\therefore W = \frac{(CV)^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

This work is stored as electrostatic potential energy of capacitor i.e.,

Electrostatic potential energy,

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

**Energy density :** Consider a parallel plate capacitor consisting of plates, each of area A, separated by a distance d. If space between the plates is filled with a medium of dielectric constant K, then

$$\text{Capacitance of capacitor, } C = \frac{K\epsilon_0 A}{d}$$

If  $\sigma$  is the surface charge density of plates, then electric field strength between the plates

$$E = \frac{\sigma}{K\epsilon_0} \Rightarrow \sigma = K\epsilon_0 E$$

Charge on each plate of capacitor,  $Q = \sigma A = K\epsilon_0 E A$

$\therefore$  Energy stored by capacitor,

$$U = \frac{Q^2}{2C} = \frac{(K\epsilon_0 EA)^2}{2(K\epsilon_0 A/d)}$$

$$= \frac{1}{2} K\epsilon_0 E^2 A d$$

But  $Ad =$  Volume of space between capacitor plates

$$\therefore \text{Energy stored, } U = \frac{1}{2} K\epsilon_0 E^2 A d$$

Electrostatic energy stored per unit volume,

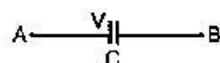
$$u_e = \frac{U}{Ad} = \frac{1}{2} K\epsilon_0 E^2$$

This is expression for electrostatic energy density

in medium of dielectric constant K.

(b) Initially, if we consider a charged capacitor, then its charge would be

$$Q = CV$$

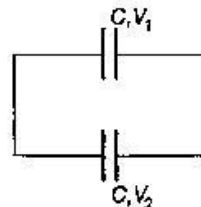


and energy stored is

$$U_1 = \frac{1}{2} CV^2 \quad \dots(i)$$

Then, this charged capacitor is connected to uncharged capacitor.

Let the common potential be V. The charge flows from first capacitor to the other capacitor unless both the capacitors attain common potential.



$$\Rightarrow Q_1 = CV_1 \text{ and } Q_2 = CV_2$$

Applying conservation of charge,

$$Q = Q_1 + Q_2 \Rightarrow CV = CV_1 + CV_2$$

$$\Rightarrow V = V_1 + V_2 \Rightarrow V_1 = \frac{V}{2} [\because V_1 = V_2]$$

Total energy stored,

$$U_2 = \frac{1}{2} CV_1^2 + \frac{1}{2} CV_2^2$$

$$U_2 = \frac{1}{2} C \left( \frac{V}{2} \right)^2 + \frac{1}{2} C \left( \frac{V}{2} \right)^2$$

$$\Rightarrow U_2 = \frac{1}{4} CV^2 \quad \dots(ii)$$

From Equations (i) and (ii), we get

$$U_2 < U_1$$

$\Rightarrow$  Energy stored in the combination is less than that stored initially in single capacitor.

25. Explain, using a labelled diagram, the principle and working of a moving coil galvanometer. What is the function of (i) uniform radial magnetic field, (ii) soft iron core ?

Define the terms (i) current sensitivity and (ii) voltage sensitivity of a galvanometer. Why does increasing the current sensitivity not necessarily increase voltage sensitivity? [5]

OR

- (a) Write using Biot-Savart law, the expression for the magnetic field  $\vec{B}$  due to an element  $d\vec{l}$  carrying current I at a distance  $\vec{r}$  from it in a vector form.



Hence derive the expression for the magnetic field due to a current carrying loop of radius R at a point P distant X from its centre along the axis of the loop.

(b) Explain how Biot-Savart law enables one to express the Ampere's circuital law in the integral form, viz.,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Where I is the total current passing through the surface.

**Answer :** The basic principle of a moving coil galvanometer is that when a current carrying coil is placed in a magnetic field, it experiences a torque.

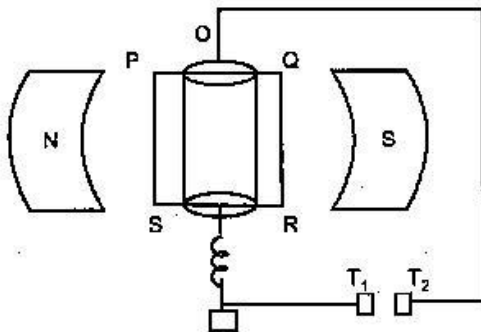
When the current I is passed through the coil, the torque experienced is given by  $\tau = NIAB \sin \theta$

Where N = No. of turns of the coil,

A = Area of the coil

B = Magnetic field and

$\theta$  = Angle between normal of coil and magnetic field



(i) The uniform radial magnetic field is used to make the scale linear.

(ii) The soft iron core increases the strength of the magnetic field.

The current sensitivity is defined as the deflection produced in the galvanometer, while passing a current of 1 ampere. (1 amp). through it

Thus, current sensitivity  $\left(\frac{\alpha}{I}\right) = \frac{NBA}{K}$ .

The voltage sensitivity is defined as the deflection produced in the galvanometer when a potential difference of 1V is applied to the coil.

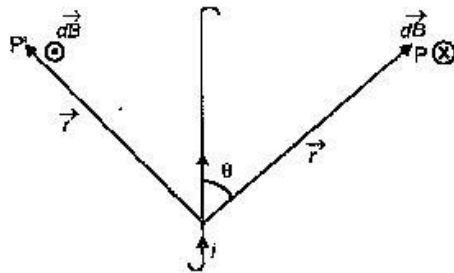
Thus, voltage sensitivity  $\left(\frac{\alpha}{V}\right) = \frac{NBA}{KR}$ .

Where R is the resistance.

Increasing the current sensitivity does not necessarily increase the voltage sensitivity as there is an increase in the resistance as well.

OR

(a) Suppose we have a conductor of length l in which current i is flowing. We need to calculate the magnetic field at a point P in vacuum. If  $i d\vec{l}$  is one of the infinitely small current element, the magnetic field  $d\vec{B}$  at point P is given by

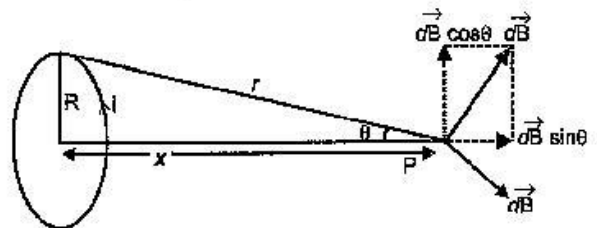


$$d\vec{B} \propto \frac{i d\vec{l} \times \vec{r}}{r^3}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3}$$

Where  $\frac{\mu_0}{4\pi}$  is a proportionality constant.

Suppose there is a circular coil of radius R, carrying a current i. Let P be a point at the axis of the coil at a distance x from the centre, at which the field is required.



Consider a conducting element dl of the loop. The magnetic field due to dl is given by the Biot-Savart law,

$$dB = \frac{\mu_0}{4\pi} \frac{i |d\vec{l} \times \vec{r}|}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{idl}{(R^2 + x^2)^{3/2}}$$

The direction of dB is perpendicular to the plane formed by dl and r. It has an X-component  $dB_x$  and a component perpendicular to X-axis,  $dB_{\perp}$ . When the components perpendicular to the X-axis are summed over, they cancel out and we obtain null result. Thus, only the X-component survives.

So the resultant field  $\vec{B}$  at P is given by

$$B = \int dB \sin \theta$$

$$B = \frac{\mu_0}{4\pi} \frac{i}{r^2} \int dl \sin \theta$$

$$B = \frac{\mu_0 i R}{4\pi r^3} \int dl \left[ \because \sin \theta = \frac{R}{r} \right]$$

But  $\int dl = 2\pi R$  and  $r = (R^2 + x^2)^{1/2}$

$$\therefore B = \frac{\mu_0 2\pi i R^2}{4\pi (R^2 + x^2)^{3/2}}$$

If the coil has  $N$  turns, then each turn will contribute equally to  $B$ . Then,

$$B = \frac{\mu_0 N i R^2}{2(x^2 + R^2)^{3/2}}$$

(b) According to Biot-Savart law the line integral of the magnetic field  $\vec{B}$  around any 'closed' path is equal to  $\mu_0$  times the net current  $I$  passing through the area enclosed by the path.

$$\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

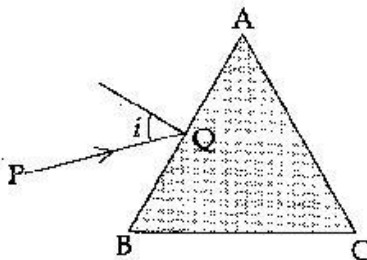
Where  $\mu_0$  is the permeability of free space. Ampere's circuital law in electromagnetism is analogous to Gauss' law in electrostatics.

26. (a) Consider two coherent sources  $S_1$  and  $S_2$  producing monochromatic waves to produce interference pattern. Let the displacement of the wave produced by  $S_1$  be given by  $Y_1 = a \cos \omega t$  and the displacement by  $S_2$  be  $Y_2 = a \cos(\omega t + \phi)$ . Find out the expression for the amplitude of the resultant displacement at a point and show that the intensity at that point will be  $I = 4a^2 \cos^2 \phi/2$ . Hence establish the conditions for constructive and destructive interference.

- (b) What is the effect on the interference fringes in Young's double slit experiment when (i) the width of the source slit is increased; (ii) the monochromatic source is replaced by a source of white light. [5]

OR

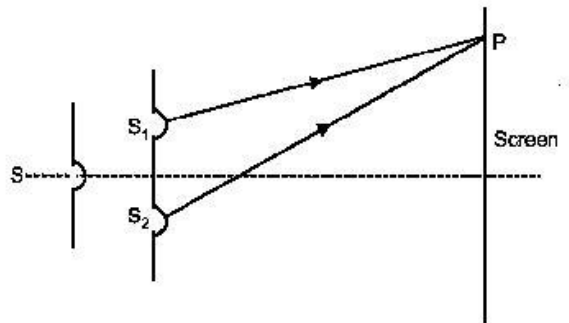
- (a) A ray 'PQ' of light is incident on the face AB of a glass prism ABC (as shown in the figure) and emerges out of the face AC. Trace the path of the ray. Show that  $\angle i + \angle e = \angle A + \angle \delta$ . When  $\delta$  and  $e$  denote the angle of deviation and angle of emergence respectively.



Plot a graph showing the variation of the angle of deviation as a function of angle of incidence. State the condition under which  $\angle \delta$  is minimum.

- (b) Find out the relation between the refractive index ( $\mu$ ) of the glass prism and  $\angle A$  for the case when the angle of prism ( $A$ ) is equal to the angle of minimum deviation ( $\delta_m$ ). Hence obtain the value of the refractive index for angle of prism  $A = 60^\circ$ .

Answer : (a) Let  $S$  be a narrow slit illuminated by a monochromatic source of light and  $S_1$  and  $S_2$  two similar parallel slits very close together and equidistant from  $S$ .



Displacement of the wave produced by  $S_1$  is given by

$$y_1 = a \cos \omega t$$

and the displacement of the wave produced by  $S_2$  is given by

$$y_2 = a \cos(\omega t + \phi)$$

The resultant displacement is given by

$$y = y_1 + y_2$$

$$y = a \cos \omega t + a \cos(\omega t + \phi)$$

$$y = 2a \cos \phi/2 \cos(\omega t + \phi/2)$$

The amplitude of the resultant displacement is  $2a \cos \phi/2$ . The intensity of light is directly proportional to the square of amplitude of the wave.

The resultant intensity is given by

$$I = 4a^2 \cos^2 \phi/2$$

For constructive interference the intensity of light at point  $P$  is maximum, if

$$\cos \phi = \max = +1 : \phi = 0, 2\pi, 4\pi, \dots$$

i.e.,  $\phi = 2n\pi$  where  $n = 0, 1, 2, \dots$

$$\begin{aligned} \text{Path difference} &= \frac{\lambda}{2\pi} \times 2n\pi \\ &= n\lambda \end{aligned}$$

For destructive interference the intensity of light at point P is minimum, if

$$\cos \phi = -1$$

$$\therefore \phi = \pi, 3\pi, 5\pi, \dots$$

$$\text{or } \phi = (2n-1)\pi \text{ where } n = 1, 2, 3, \dots$$

The corresponding path difference between the two waves

$$x = \frac{\lambda}{2\pi} \phi = \frac{\lambda}{2\pi} (2n-1)\pi = (2n-1) \frac{\lambda}{2}$$

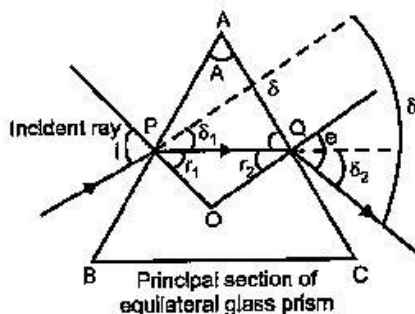
(b) (i) As the width of the slits is increased, the fringe width decreases, because,

$$\beta \propto \frac{1}{d}$$

(ii) The different colours of white light will produce different interference patterns but the central bright fringes due to all colours are at the same positions. Therefore, the central bright fringe is white in colour. Since the wavelength of the blue light is smallest, the fringe closed on the either side of the central white fringe is blue and farthest is red. Beyond a few fringes, no clear fringe pattern is visible.

OR

(a) Let the incident ray meet refracting face AB of the prism at point P. Ray PQ is the refracted ray inside the prism and  $\delta_1$  and  $r_1$  are the angle of the deviation and refraction at interface AB. At interface AC the ray goes out of the prism. Let  $e$  be the angle of emergence. The angle of deviation at point Q is  $\delta_2$  as shown in figure.



Using geometry, we see that at point P,

$$i = \delta_1 + r_1 \quad \therefore \delta_1 = i - r_1$$

and at point Q,  $e = \delta_2 + r_2$

$$\delta_2 = e - r_2$$

The total deviation  $\delta$ , suffered by the incident ray is equal to  $\delta_1 + \delta_2$ .

or  $\delta = \delta_1 + \delta_2$

$$\begin{aligned} &= (i - r_1) + (e - r_2) \\ &= (i + e) - (r_1 + r_2) \quad \dots(i) \end{aligned}$$

In quadrilateral POQA, the sum of all four angle is  $360^\circ$ .

$$P + O + Q + A = 360^\circ$$

as P and Q both are right angles

$$P + Q = 180^\circ$$

$$O + A = 180^\circ \quad \dots(ii)$$

In triangle POQ

$$O + r_1 + r_2 = 180^\circ \quad \dots(iii)$$

Comparing equations (ii) and (iii), we have

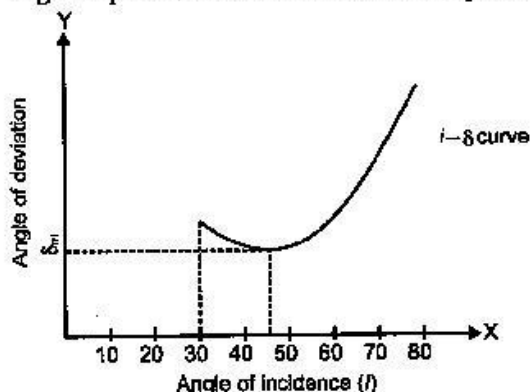
$$A = r_1 + r_2$$

Substituting this value in equation (i)

$$\delta = i + e - A$$

$$\delta + A = i + e \quad \dots(iv)$$

So angle of deviation produced by a prism depends upon the angle of incidence, refracting angle of prism, and the material of the prism.



When prism is in the position of minimum deviation,

then  $i = e$  and  $r_1 = r_2$

From equation (iv),

$$\delta_m + A = i + i$$

$$i = \frac{(\delta_m + A)}{2}$$

(b) by Snell's law,  $\mu = \frac{\sin i}{\sin r}$  putting the value of  $i$  and  $r$

$$\mu = \frac{\sin \left( \frac{A + \delta_{\min}}{2} \right)}{\sin \frac{A}{2}}$$

But  $A = \delta_m$

$$\therefore \mu = \frac{\sin \left( \frac{A + A}{2} \right)}{\sin \frac{A}{2}}$$

$$= \frac{\sin A}{\sin \frac{A}{2}} = \frac{2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}}{\sin \frac{A}{2}} = 2 \cos \frac{A}{2}$$

If  $A = 60^\circ$ ,

$$\mu = 2 \cos \frac{60^\circ}{2} = 2 \cos 30^\circ$$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

Note: All questions of Outside Delhi Set II are from Outside Delhi Set I and Outside Delhi Set III are from Set I and Set II.

SECTION-A

1. Define capacitor reactance. Write its S.I. units. [1]

Answer : Capacitor reactance is the resistance offered by a capacitor to the flow of a.c. It is given by

$$X_C = \frac{1}{\omega C}$$

Where  $\omega = 2\pi f$ ,  $f$  = frequency of the source

$$X_C = \frac{1}{2\pi f C}$$

$\omega$  = Angular frequency of the source

$C$  = Capacitance of the capacitor.

The SI unit of capacitor reactance is ohm ( $\Omega$ ).

2. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? [1]

Answer : The electric flux through a cube of side 1 cm which encloses an electric dipole will be zero, as net charge enclosed by a cube is zero.

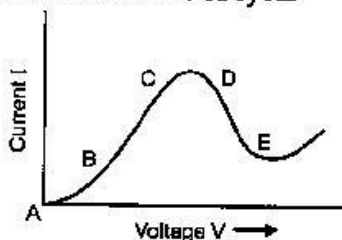
3. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? [1]

Answer : Since  $\mu_g$  lens  $<$   $\mu_m$  surroundings. It behaves like a converging lens.

4. How are side bands produced? [1]

5. Graph showing the variation of current versus voltage for a material GaAs is shown in the figure. Identify the region of: [1]

- (i) negative resistance  
(ii) where Ohm's law is obeyed.



Answer : (i) DE is the region of negative resistance because the slope of curve in this part is negative.  
(ii) BC is the region where Ohm's law is obeyed because in this part, the current varies linearly with the voltage.

SECTION-B

6. A proton and an  $\alpha$ -particle have the same de-Broglie wavelength. Determine the ratio of (i) their accelerating potentials (ii) their speeds.

Answer : (i) The de-Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

where,  $V$  is the accelerating potential of the particle.

It is given that

$$\lambda_{\text{proton}} = \lambda_{\text{alpha}}$$

$$\Rightarrow \frac{h}{\sqrt{2m_p q_p V_p}} = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$$

$$\sqrt{2m_\alpha q_\alpha V_\alpha} = \sqrt{2m_p q_p V_p}$$

On squaring both sides, we get

$$2m_\alpha q_\alpha V_\alpha = 2m_p q_p V_p$$

$$\Rightarrow \frac{V_p}{V_\alpha} = \frac{m_\alpha q_\alpha}{m_p q_p} = \frac{4m_p \times 2q_p}{m_p q_p} \left[ \begin{array}{l} \because m_\alpha = 4m_p \text{ and} \\ q_\alpha = 2q_p \end{array} \right]$$

$$= \frac{8}{1}$$

(ii) We can also write de-Broglie wavelength as

$$\lambda = \frac{h}{2\pi m v}$$

where  $h$  is Planck's constant,  $m$  is mass of the particle and  $v$  is speed of the particle.

It is given that

$$\lambda_{\text{proton}} = \lambda_{\text{alpha}}$$

We know

$$m_{\text{alpha}} = 4m_{\text{proton}}$$

$$\therefore \lambda_{\text{alpha}} = \frac{h}{4m_{\text{proton}} v_{\text{alpha}}}$$

$$\Rightarrow \text{Now } \frac{h}{m_{\text{proton}} v_{\text{proton}}} = \frac{h}{4m_{\text{proton}} v_{\text{alpha}}}$$

$$\frac{v_{\text{proton}}}{v_{\text{alpha}}} = 4$$

7. Show that the radius of the orbit in hydrogen atom varies as  $n^2$ . Where  $n$  is the principal quantum number of the atom. [2]

Answer : According to the Bohr's theory of hydrogen atom, the angular momentum of a revolving electron is given by

$$mvr = \frac{nh}{2\pi}$$

...(i)

- Here,  $m$  = Mass of the electron  
 $v$  = Velocity of the electron  
 $r$  = Radius of the orbit  
 $h$  = Planck's constant  
 $n$  = Principal quantum number of the atom

If an electron of mass  $m$  and velocity  $v$  is moving in a circular orbit of radius  $r$ , then the centripetal force required is given by

$$F = \frac{mv^2}{r}$$

Also, if the charge on the nucleus is  $Ze$ , then the force of electrostatic attraction between the nucleus and the electron will provide the necessary centripetal force.

$$F = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r^2} = \frac{KZe^2}{r^2}$$

where  $K = \frac{1}{4\pi\epsilon_0}$

$$\therefore \frac{mv^2}{r} = \frac{KZe^2}{r^2} \quad \dots(ii)$$

From (i), we get

$$v = \frac{nh}{2\pi mr}$$

Putting this value in (ii), we get

$$\frac{m}{r} \frac{n^2 h^2}{4\pi^2 m^2 r^2} = \frac{KZe^2}{r^2}$$

$$\Rightarrow r = \frac{n^2 h^2}{4\pi^2 m K Z e^2}$$

$$\Rightarrow r \propto n^2.$$

8. Distinguish between 'intrinsic' and 'extrinsic' semiconductors. [2]

Answer :

Intrinsic semiconductor	Extrinsic semiconductor
1. It is pure semiconducting material with no impurity atoms added to it.	It is prepared by doping a small quantity of impurity atoms to the pure semiconductor.
2. The number of free electrons in the conduction band and the number of holes in valence band is exactly equal.	The number of free electrons and holes is never equal. There is an excess of electrons in $n$ -type semiconductors and an excess of holes $p$ -type semiconductors.

3. Its electrical conductivity is a function of temperature alone.	Its electrical conductivity depends upon the temperature and the amount of impurity added in them. to it.
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9. Use the mirror equation to show that an object placed between  $f$  and  $2f$  of a concave mirror produces a real image beyond  $2f$ . [2]

OR

Find an expression for intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum ?

Answer :

Mirror equation is

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where  $u$  is the distance of object from the mirror,  $v$  is the distance of image from the mirror and  $f$  is the focal length of the mirror.

For a concave mirror  $f$  is negative i.e.  $f < 0, u < 0$

For a real object (on the left of mirror)

$$\therefore 2f < u < f$$

$$\Rightarrow \frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

$$\frac{1}{2f} - \frac{1}{f} > \frac{1}{u} - \frac{1}{f} > \frac{1}{f} - \frac{1}{f}$$

$$\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f}$$

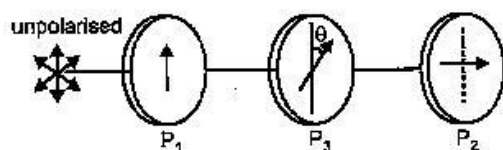
$$\Rightarrow \frac{1}{2f} < \frac{1}{v} < 0$$

$$\Rightarrow v < 0. \text{ Also } v > 2f$$

$\therefore$  Image is real.

This implies that  $v$  is negative and greater than  $2f$ . This means that the image lies beyond  $2f$ .

OR



Let us consider two crossed polarisers  $P_1$  and  $P_2$  with a polaroid sheet  $P_3$  placed between them.

Let  $I_0$  be the intensity of polarised light after passing through the first polarizer  $P_1$ . If  $\theta$  is

the angle between the axes of  $P_1$  and  $P_3$ , then the intensity of the polarized light after passing through  $P_3$  will be  $I = I_0 \cos^2 \theta$ .

As  $P_1$  and  $P_2$  are crossed. The angle between the axes of  $P_1$  and  $P_2 = 90^\circ$ .

$\therefore$  Angle between the axes of  $P_2$  and  $P_3 = (90^\circ - \theta)$

The intensity of light emerging from  $P_2$  will be given by

$$\begin{aligned} I' &= I \cos^2 (90 - \theta) \\ I' &= [I_0 \cos^2 \theta] \cos^2 (90^\circ - \theta) \\ \Rightarrow I' &= [I_0 \cos^2 \theta] \sin^2 \theta \\ \Rightarrow I' &= \frac{I_0}{4} (4 \cos^2 \theta \sin^2 \theta) \\ \Rightarrow I' &= \frac{I_0}{4} (2 \sin \theta \cos \theta)^2 \\ \Rightarrow I' &= \frac{I_0}{4} (\sin^2 2\theta) \end{aligned}$$

The intensity of polarized light transmitted from  $P_2$  will be maximum when

$$\begin{aligned} \sin 2\theta &= \text{maximum} = 1 \\ \Rightarrow \sin 2\theta &= \sin 90^\circ \\ \Rightarrow 2\theta &= 90^\circ \\ \Rightarrow \theta &= 45^\circ \end{aligned}$$

The maximum transmitted intensity of polarised light is

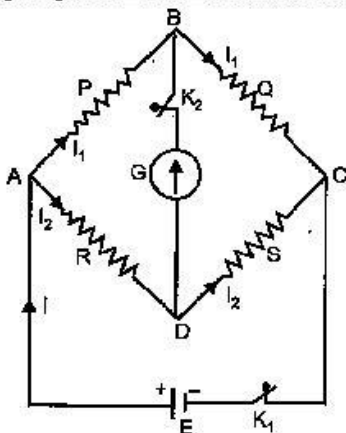
$$I' = \frac{I_0}{4}$$

10. Use Kirchhoff's rules to obtain conditions for the balance condition in a Wheatstone bridge. [2]

**Answer :** Let us consider a Wheatstone bridge arrangement as shown below :

Wheatstone bridge is a special bridge type circuit which consists of four resistances, a galvanometer and a battery. It is used to determine unknown resistance.

In figure four resistance  $P$ ,  $Q$ ,  $R$  and  $S$  are connected in the form of four arms of a quadrilateral. Let the current given by battery in the balanced position be  $I$ . This current on reaching point  $A$  is divided into two parts  $I_1$  and  $I_2$ . As there is no current in galvanometer in balanced state, therefore, current in resistances  $P$  and  $Q$  is  $I_1$  and in resistances  $R$  and  $S$  it is  $I_2$ .



Applying Kirchhoff's first law i.e. junction law at point A

$$I - I_1 - I_2 = 0 \text{ or } I = I_1 + I_2 \quad \dots(i)$$

Applying Kirchhoff's second law to closed mesh ABDA

$$- I_1 P + I_2 R = 0 \text{ or } I_1 P = I_2 R \quad \dots(ii)$$

Applying Kirchhoff's second law to mesh BCDB

$$- I_1 Q + I_2 S = 0 \text{ or } I_1 Q = I_2 S \quad \dots(iii)$$

Dividing equation (ii) by (iii), we get

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S} \quad \text{or} \quad \frac{P}{Q} = \frac{R}{S}$$

This is condition of balance Wheatstone's bridge.

### SECTION-C

11. Name the parts of the electromagnetic spectrum which is

(a) suitable for radar systems used in aircraft navigation

(b) used to treat muscular strain

(c) used as a diagnostic tool in medicine

Write in brief, how these waves can be produced.

**Answer :** (a) Microwaves are suitable for radar systems that are used in aircraft navigation.

These rays are produced by special vacuum tubes, namely klystrons, magnetrons and Gunn diodes.

(b) Infrared waves are used to treat muscular strain.

These rays are produced by hot bodies and molecules.

(c) X-rays are used as a diagnostic tool in medicine.

These rays are produced when high energy electrons are stopped suddenly on a metal of high atomic number.

12. (i) A giant refracting telescope has an objective lens of focal length 15 m. If an eye-piece of focal length 1.0 cm is used. What is the angular magnification of the telescope ?

(ii) If this telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective lens ? The diameter of the moon is  $3.48 \times 10^6$  m and the radius of lunar orbit is  $3.8 \times 10^8$  m. [3]

**Answer :** (i) Let

$f_o$  = Focal length of the objective lens = 15 m = 1500 cm

$f_e$  = Focal length of the eye lens = 1.0 cm

Angular magnification of the giant refracting telescope is given by

$$m_0 = \left| \frac{f_o}{f_e} \right|$$

$$m_0 = \left| \frac{1500}{1} \right| = 1500$$

(ii) Diameter of the image of the moon formed by the objective lens,  $d = \alpha f_0$

$$\Rightarrow d = \frac{\text{Diameter of the moon}}{\text{Radius of the lunar orbit}} \times f_0$$

$$\Rightarrow d = \frac{3.48 \times 10^6}{3.8 \times 10^8} \times 1500 \times 10^{-2}$$

$$\Rightarrow d = 0.1374 \text{ m.}$$

$$d = 13.74 \text{ cm}$$

13. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.

The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from  $\lambda_1$  to  $\lambda_2$ . Derive the expressions for the threshold wavelength  $\lambda_0$  and work function for the metal surface. [3]

Answer : Einstein's photoelectric equation is given by

$$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2$$

$$K_{\text{max}} = h\nu - \phi_0$$

$$\text{or} \quad h\nu = h\nu_0 + \frac{1}{2} m v_{\text{max}}^2$$

where

$K_{\text{max}}$  = Maximum kinetic energy of the photoelectron

$v_{\text{max}}$  = Maximum velocity of the emitted photoelectron

$m$  = Mass of the photoelectron

$\nu$  = Frequency of the light radiation

$\phi_0$  = Work function

$h$  = Planck's constant

If  $\nu_0$  is the threshold frequency, then the work function can be written as

$$W = \phi_0 = h\nu_0$$

$$\Rightarrow K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = h\nu - h\nu_0 = h(\nu - \nu_0)$$

The above equations explains the following results :

1. If  $\nu < \nu_0$ , then the maximum kinetic energy is negative, which is impossible. Hence, photoelectric emission does not take place for the incident radiation below the threshold frequency. Thus, the photoelectric emission can take place if  $\nu > \nu_0$ .

2. The maximum kinetic energy of emitted photoelectrons is directly proportional to the

frequency of the incident radiation. This means that maximum kinetic energy of photoelectron depends only on the frequency of incident light not on the intensity. According to the photoelectric equation,

$$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = h\nu - \phi_0$$

$$K_{\text{max}} = \frac{hc}{\lambda_1} - \phi_0 \quad \dots(i)$$

Let the maximum kinetic energy for the wavelength  $\lambda_2$  be  $K_2$ .

$$K_2 = \frac{hc}{\lambda_2} - \phi_0 \quad \dots(ii)$$

$$K_2 = 2K_1 \quad (\text{given})$$

From (i) and (ii), we have

$$\frac{hc}{\lambda_2} - \phi_0 = 2 \left( \frac{hc}{\lambda_1} - \phi_0 \right)$$

$$\Rightarrow \phi_0 = hc \left( \frac{2}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$\Rightarrow h\nu_0 = hc \left( \frac{2}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$\Rightarrow \frac{c}{\lambda_0} = c \left( \frac{2}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$\Rightarrow \frac{1}{\lambda_0} = \left( \frac{2}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$\Rightarrow \lambda_0 = \left( \frac{\lambda_1 \lambda_2}{2\lambda_2 - \lambda_1} \right)$$

Work function is the energy required to eject a photoelectron from the metal.

$$W = \frac{hc}{\lambda_0}$$

$$\therefore W = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}$$

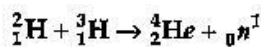
14. In the study of Geiger-Marsden experiment on scattering of  $\alpha$ -particles by a thin foil of gold, draw the trajectory of  $\alpha$ -particles in the Coulomb field of target nucleus. Explain briefly how one gets the information on the size of the nucleus from this study.

From the relation  $R = R_0 A^{1/3}$ , where  $R_0$  is constant and  $A$  is the mass number of the nucleus, show that nuclear matter density is independent of  $A$ .

[3]

OR

Distinguish between nuclear fission and fusion. Show how in both these processes energy is released. Calculate the energy release in MeV in the deuterium-tritium fusion reaction :



Using the data :

$$m({}^2_1\text{H}) = 2.014102 \text{ u}$$

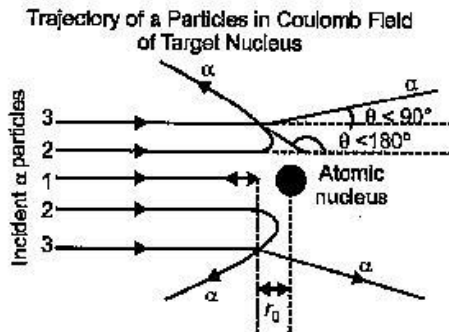
$$m({}^3_1\text{H}) = 3.016049 \text{ u}$$

$$m({}^4_2\text{He}) = 4.002603 \text{ u}$$

$$m_n = 1.008665 \text{ u}$$

$$1 \text{ u} = 931.5 \text{ MeV}/c^2$$

Answer :



From this experiment, the following is observed :

1. Most of the alpha particles pass straight through the gold foil. It means that they do not suffer any collision with gold atoms.

2. About one alpha particle in every 8000 alpha particles deflects by more than  $90^\circ$ .

As most of the alpha particles go undeflected and only a few get deflected, this shows that most of the space in an atom is empty and at the centre of the atom, there exists a nucleus. By the number of the alpha particles deflected, the information regarding size of the nucleus can be known.

If  $m$  is the average mass of a nucleon and  $R$  is the nuclear radius, then mass of nucleus =  $mA$ , where  $A$  is the mass number of the element.

$$\text{Volume of the nucleus, } V = \frac{4}{3}\pi R^3$$

$$\text{Given } R = R_0 A^{1/3}$$

$$\Rightarrow V = \frac{4}{3}\pi (R_0 A^{1/3})^3$$

$$\Rightarrow V = \frac{4}{3}\pi R_0^3 A$$

$$\text{Density of nuclear matter, } \rho = \frac{mA}{V}$$

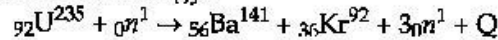
$$\Rightarrow \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3}$$

This shows that the nuclear density is independent of  $A$ .

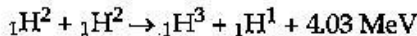
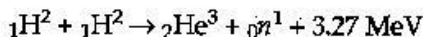
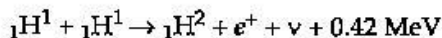
OR

**Nuclear fission:** Nuclear fission is a disintegration process, in which a heavier nucleus gets split up into two lighter nuclei, with the release of a large amount of energy.

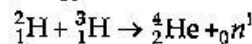


Here, the energy released per fission of  ${}_{92}\text{U}^{235}$  is 200.4 MeV.

**Nuclear fusion :** When two or more light nuclei combine to form a heavy stable nuclide, part of mass disappears in the process and is converted into energy. This phenomenon is called nuclear fusion.



Energy released in this process



$$\therefore \Delta m = (2.014102 + 3.016049) - (4.002603 + 1.008665)$$

$$= 0.018883 \text{ u}$$

$$\text{Energy released, } Q = 0.018883 \times 931.5 \text{ MeV}/c^2 = 17.589 \text{ MeV}$$

15. Draw a block diagram of a detector for AM signal and show, using necessary processes and the waveforms, how the original message signal is detected from the input AM wave.\*\* [3]

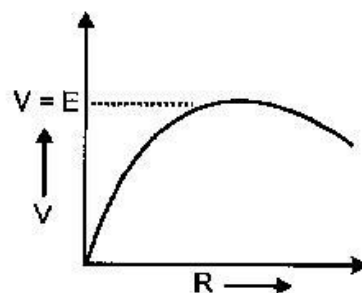
Answer :

16. A cell of emf 'E' and internal resistance 'r' is connected across a variable load resistor R. Draw the plots of the terminal voltage V versus (i) R and (ii) the current i.

It is found that when  $R = 4 \Omega$ , the current is 1 A when R is increased to  $9 \Omega$ , the current reduces to 0.5 A. Find the values of the emf E and internal resistance r. [3]

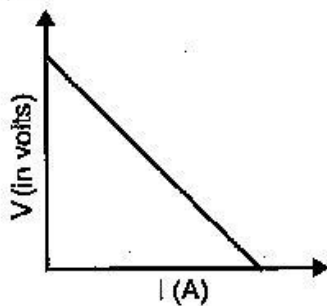
Answer :

- (i) Graph between terminal voltage (V) and resistance (R) :





(ii) Graph between terminal voltage ( $V$ ) and current ( $I$ ):



(iii) When  $R = 4 \Omega$  and  $I = 1 \text{ A}$

We know,

Terminal voltage,  $V = E - Ir$

So, we have

$$V = IR = 4 = E - Ir$$

$$E - r = 4 \quad \dots (i)$$

When  $R = 9 \Omega$  and  $I = 0.5 \text{ A}$

$$V = IR = 0.5 \times 9 = E - 0.5r$$

$$E - 0.5r = 4.5 \quad \dots (ii)$$

Subtracting (i) from (ii), we get

$$E - 0.5r - E + r = 4.5 - 4$$

$$0.5r = 0.5$$

$$r = 1 \Omega$$

Substituting value of  $r$  in (i)

$$E - 1 = 4$$

$$E = 5 \text{ V}$$

Thus  $r = 1 \Omega$  and  $E = 5 \text{ V}$ .

17. Two capacitors of unknown capacitances  $C_1$  and  $C_2$  are connected first in series and then in parallel across a battery of  $100 \text{ V}$ . If the energy stored in the two combinations is  $0.045 \text{ J}$  and  $0.25 \text{ J}$  respectively, determine the value of  $C_1$  and  $C_2$ . Also calculate the charge on each capacitor in parallel combination. [3]

**Answer :** When the capacitors are connected in parallel.

Equivalent capacitance,  $C_p = C_1 + C_2$

The energy stored in the combination of the capacitors,

$$E_p = \frac{1}{2} C_p V^2$$

$$\Rightarrow E_p = \frac{1}{2} (C_1 + C_2) (100)^2 = 0.25 \text{ J}$$

$$\Rightarrow (C_1 + C_2) = 5 \times 10^{-5} \quad \dots (i)$$

When the capacitors are connected in series.

Equivalent capacitance,

$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$

The energy stored in the combination of the capacitors,

$$E_s = \frac{1}{2} C_s V^2$$

$$\Rightarrow E_s = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (100)^2 = 0.045 \text{ J}$$

$$\Rightarrow \frac{1}{2} \frac{C_1 C_2}{5 \times 10^{-5}} (100)^2 = 0.045 \text{ J} \quad [\because C_1 + C_2 = 5 \times 10^{-5}]$$

$$\Rightarrow C_1 C_2 = 0.045 \times 10^{-4} \times 5 \times 10^{-5} \times 2 = 4.5 \times 10^{-10} \quad (ii)$$

$$(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1 C_2$$

$$\Rightarrow (C_1 - C_2) = \sqrt{7 \times 10^{-10}} = 2.64 \times 10^{-5} \quad [\text{using (i) \& (ii)}]$$

$$\text{Now, } C_1 - C_2 = 2.64 \times 10^{-5} \quad \dots (ii)$$

Solving (i) and (iii), we get

$$C_1 = 38.2 \mu\text{F} \text{ and } C_2 = 11.8 \mu\text{F}$$

When the capacitors are connected in parallel, the charge on each of them can be obtained as follows :

$$Q_1 = C_1 V = 38.2 \times 10^{-6} \times 100 = 38.2 \times 10^{-4} \text{ C}$$

$$Q_2 = C_2 V = 11.8 \times 10^{-6} \times 100 = 11.8 \times 10^{-4} \text{ C}$$

18. State the principle of working of a galvanometer. A galvanometer of resistance  $G$  is converted into a voltmeter to measure upto  $V$  volts by connecting a resistance  $R_1$  in series with the coil. If a resistance  $R_2$  is connected in series with it, then it can measure upto  $V/2$  volts. Find the resistance, in terms of  $R_1$  and  $R_2$ , required to be connected to convert it into a voltmeter that can read upto  $2 \text{ V}$ . Also find the resistance  $G$  of the galvanometer in terms of  $R_1$  and  $R_2$ . [3]

**Answer :** Principle : When a current-carrying coil is placed in a magnetic field, it experiences a torque. From the measurement of the deflection of the coil, the strength of the current can be computed. A high resistance is connected in series with the galvanometer to convert it into voltmeter. The value of the resistance is given by

$$R = \frac{V}{I_g} - G$$

Here,

$V$  = Potential difference across the terminals of the voltmeter

$I_g$  = Current through the galvanometer

$G$  = Resistance of the galvanometer

When the resistance  $R_1$  is connected in series with the galvanometer.

$$V = I_g (G + R_1)$$

When the resistance  $R_2$  is connected in series with the galvanometer<sup>7</sup>

$$\frac{V}{2} = I_g (G + R_2)$$

$$\Rightarrow 2 = \frac{G + R_1}{G + R_2}$$

$$G = R_1 - 2R_2$$

Let  $R_3$  be the resistance required for conversion of galvanometer into voltmeter of range 2V

$$\therefore 2V = I_g (G + R_3)$$

Also  $V = I_g (G + R_1)$

$$\therefore 2 = \frac{G + R_3}{G + R_1}$$

$$\therefore R_3 = G + 2R_1 = R_1 - 2R_2 + 2R_1 = 3R_1 - 2R_2$$

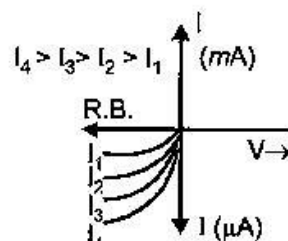
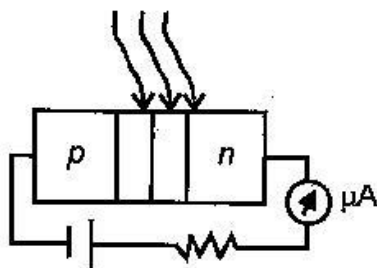
19. With what considerations in view, a photodiode is fabricated? State its working with the help of a suitable diagram.

Even though the current in the forward bias is known to be more than in the reverse bias, yet the photodiode works in reverse bias. What is the reason? [3]

Answer : A photodiode is used to observe the change in current with change in the light intensity under reverse bias condition.

In fabrication of photodiode, material chosen should have band gap  $\sim 1.5$  eV or lower so that solar conversion efficiency is better. This is the reason to choose Si or GaAs material.

**Working :** It is a  $p-n$  junction fabricated with a transparent window to allow light photons to fall on it. These photons generate electron hole pairs upon absorption. If the junction is reverse biased using an electrical circuit, these electron hole pair move in opposite directions so as to produce current in the circuit. This current is very small and is detected by the microammeter placed in the circuit.



I-V characteristics of photodiode for different illumination intensities

A photodiode is preferably operated in reverse bias condition. Consider an  $n$ -type semiconductor. Its majority carrier (electron) density is much larger than the minority hole density i.e.  $n \gg p$ . When illuminated with light, both types of carriers increase equally in number

$$n' = n + \Delta n; p' = p + \Delta p$$

Now  $n \gg p$  and  $\Delta n = \Delta p$

$$\frac{\Delta n}{n} \ll \frac{\Delta p}{p}$$

That is, the fractional increase in majority carriers is much less than the fractional increase in minority carriers. Consequently, the fractional change due to the photo-effects on the minority carrier dominated reverse bias current is more easily measurable than the fractional change in the majority carrier dominated forward bias current. Hence, photodiodes are preferably used in the reverse bias condition for measuring light intensity.

20. Draw a circuit diagram of a transistor amplifier in CE configuration.

Define the terms :

(i) Input resistance and (ii) Current amplification factor. How are these determined using typical input and output characteristics? [3]

21. Answer the following questions : [3]

- (a) In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is  $0.1^\circ$ . Find the spacing between the two slits.
- (b) Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected?

Answer : (a) Angular width ( $\theta$ ) of fringe in double-slit experiment is given by  $\theta = \frac{\lambda}{d}$

Where  $d$  = Spacing between the slits.

Given : Wavelength of light,  $\lambda = 600$  nm

Angular width of fringe,

$$\theta = 0.1^\circ = 0.1 \times \frac{\pi}{180} \text{ rad} = 0.0018 \text{ rad}$$

### SECTION-D

$$\therefore d = \frac{\lambda}{\theta}$$

$$d = \frac{600 \times 10^{-9}}{18 \times 10^{-4}} = 33.33 \times 10^{-5}$$

$$\Rightarrow d = 0.33 \times 10^{-3} \text{ m}$$

(b) The frequency and wavelength of reflected wave will not change.

The refracted wave will have same frequency, only wavelength will change.

The velocity of light in water is given by

$$v = \lambda f$$

where,  $v$  = Velocity of light

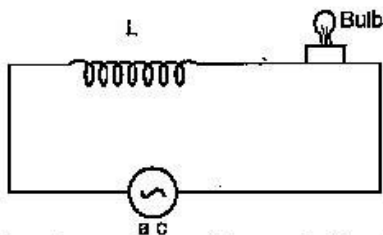
$f$  = Frequency of light

$\lambda$  = Wavelength of light

As light ray in travelling from rarer (air) medium to denser medium, its speed will decrease. Hence wavelength ( $\lambda$ ) will also decrease.

22. An inductor  $L$  of inductance  $X_L$  is connected in series with a bulb  $B$  and an ac source. How would brightness of the bulb change when
- number of turn in the inductor is reduced.
  - an iron rod is inserted in the inductor and
  - a capacitor of reactance  $X_C = X_L$  is inserted in series in the circuit. Justify your answer in each case. [3]

**Answer :**



(i) When the number of turns in the inductor is reduced, its reactance  $X_L$  decreases. The current in the circuit increases and hence brightness of the bulb increases.

(ii) When an iron rod is inserted in the inductor, the self inductance increases. Consequently, the inductive reactance  $X_L = \omega L$  increases. This decreases the current in the circuit and the bulb glows dimmer.

(iii) With capacitor of reactance  $X_C = X_L$ , the impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

becomes minimum, the current in circuit becomes maximum. Hence, the bulb glows with maximum brightness.

23. A group of students while coming from the school noticed a box marked "Danger H.T. 2200 V" at a substation in the main street. They did not understand the utility of such a high voltage, while they argued the supply was only 220 V. They asked their teacher this question the next day. The teacher thought it to be an important question and therefore, explained to the whole class. [4]

Answer the following questions :

- What device is used to bring the high voltage down to low voltage of a.c. current and what is the principle of its working?
- Is it possible to use this device for bringing down the high dc voltage to the low voltage? Explain.
- Write the values displayed by the students and the teacher.\*\*

**Answer :** (i) The device that is used to bring high voltage down to low voltage of an a.c. current is a transformer. It works on the principle of mutual induction of two windings or circuits. When current in one circuit changes, emf is induced in the neighbouring circuit.

(ii) The transformer cannot convert d.c. voltages because it works on the principle of mutual induction. When the current linked with the primary coil changes the magnetic flux linked with the secondary coil also changes. This change in flux induces emf in the secondary coil. If we apply a direct current to the primary coil the current will remain constant. Thus, there is no mutual induction and hence no emf is induced.

### SECTION-E

24. (a) State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius, having 'n' turns per unit length and carrying a steady current  $I$ .
- (b) An observer to the left of a solenoid of  $N$  turns each of cross section area  $A$  observes that a steady current  $I$  in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic momentum  $M = NIA$ . [5]



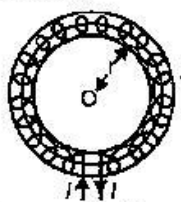
OR

- Define mutual inductance and write its S.I. units.
- Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.

(c) In an experiment two coils  $c_1$  and  $c_2$  are placed close to each other. Find out the expression for the emf induced in the coil  $c_1$  due to a change in the current through the coil  $c_2$ .

**Answer :** (a) Ampere's circuital law in electromagnetism is analogous to Gauss' law in electrostatics. This law states that "The line integral of resultant magnetic field along a closed plane curve is equal to  $\mu_0$  time the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$  where  $\mu_0$  is permeability of free space and  $I_{enc}$  is the net current enclosed by the loop.

A toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. Consider an air-cored toroid (as shown below) with centre O.

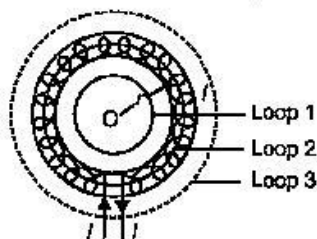


Given :  $r$  = Average radius of the toroid

$I$  = Current through the solenoid

$n$  = Number of turns per unit length

To determine the magnetic field inside the toroid, we consider three amperian loops (loop 1, loop 2 and loop 3) as shown in the figure below.



According to Ampere's circuital law, we have

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \text{ (Total current)}$$

Total current for loop 1 is zero because no current is passing through this loop

So, for loop 1

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \text{ (Total current)}$$

For loop 3

According to Ampere's circuital law, we have

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \text{ (Total current)}$$

Total current for loop 3 is zero because net current coming out of this loop is equal to the net current going inside the loop.

For loop 2 :

The total current flowing through the toroid is  $NI$ , where  $N$  is the total number of turns

$$\oint \vec{B} \cdot d\vec{l} = 0 = \mu_0 (NI) \quad \dots(i)$$

Now,  $\vec{B}$  and  $d\vec{l}$  are in the same direction

$$\oint \vec{B} \cdot d\vec{l} = B \oint dl$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = B(2\pi r) \quad \dots(ii)$$

Comparing (i) and (ii), we get

$$B(2\pi r) = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

Number of turns per unit length is given by

$$n = \frac{N}{2\pi r}$$

$$\therefore B = \mu_0 n I$$

This is the expression for magnetic field inside air-cored toroid.

(b) Given that the current flows in the clockwise direction for an observer on the left side of the solenoid. This means that left face of the solenoid acts as south pole and right face acts as north pole. Inside a bar magnet the magnetic field lines are directed from south to north. Therefore, the magnetic field lines are directed from left to right in the solenoid.

Magnetic moment of single current carrying loop is given by  $m' = IA$

where  $I$  = Current flowing through the loop

$A$  = Area of the loop

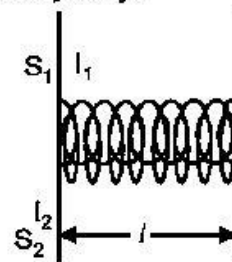
So, Magnetic moment of the whole solenoid is given by

$$M = Nm' = N(IA)$$

OR

(a) Mutual inductance is the property of two coils by the virtue of which each opposes any change in the value of current flowing through the other by developing an induced emf. The SI unit of mutual inductance is henry and its symbol is H.

(b) Consider two long solenoids  $S_1$  and  $S_2$  of same length  $l$  such that solenoid  $S_2$  surrounds solenoid  $S_1$  completely.



Let :

$n_1$  = Number of turns per unit length of  $S_1$

$n_2$  = Number of turns per unit length of  $S_2$

$I_1$  = Current passed through solenoid  $S_1$

$\phi_{21}$  = Flux linked with  $S_2$  due to current flowing through  $S_1$

$$\phi_{21} \propto I_1$$

$$\phi_{21} = M_{21}I_1$$

where  $M_{21}$  = Coefficient of mutual induction of the two solenoids

When current is passed through solenoid  $S_1$ , an emf is induced in solenoid  $S_2$ .

Magnetic field produced inside solenoid  $S_1$  on passing current through it is given by

$$B_1 = \mu_0 n_1 I_1$$

Magnetic flux linked with each turn of solenoid  $S_2$  will be equal to  $B_1$  times the area of cross-section of solenoid  $S_1$ .

Magnetic flux linked with each turn of the solenoid  $\phi_{21} = B_1 A$

Therefore, total magnetic flux linked with the solenoid  $S_2$  is given by

$$\begin{aligned} \phi_{21} &= B_1 A = \mu_0 n_1 I_1 A \\ M_{21} &= \frac{N_2 \phi_{21}}{I_1} \\ M_{21} &= \frac{N_2 \mu_0 n_1 I_1 A}{I_1} = \mu_0 n_1 N_2 A \end{aligned}$$

where  $N_2$  is total number of turns wound over the secondary coil.

$$\therefore M_{21} = \mu_0 n_1 N_2 A$$

Similarly the mutual inductance between the two solenoids when current is passed through solenoid  $S_2$  and induced emf is produced in solenoid  $S_1$  is given by

$$M_{12} = \mu_0 N_1 n_2 A$$

where  $N_1$  total numbers of turns wound over primary coil.

(c) It is found that,

$$\phi \propto I$$

$$\phi = MI$$

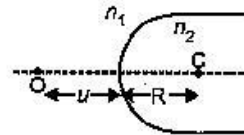
Where,  $I$  is the strength of current in coil 2, and  $\phi$  is the total amount of magnetic flux linked with coil 1.

E.m.f. induced in neighbouring coil  $C_1$  is,

$$\begin{aligned} e &= -\frac{d\phi}{dt} \\ &= -\frac{d(MI)}{dt} = -M \frac{dI}{dt} \end{aligned}$$

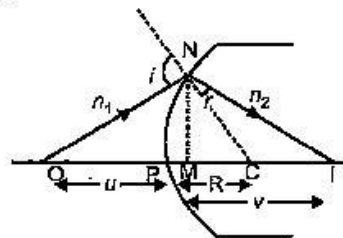
25. (a) A point object 'O' is kept in a medium of refractive index  $n_1$  in front of a convex spherical surface of radius of curvature  $R$  which separate the second medium of refractive index  $n_2$  from the first one as shown in the figure.

Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of  $n_1$ ,  $n_2$  and  $R$ .



- (b) When the image formed above acts as a virtual object for a concave spherical surface separating the medium  $n_2$  from  $n_1$  ( $n_2 > n_1$ ) draw this ray diagram and write the similar (similar to (a)) relation. Hence obtain the expression for the Lens Maker's formula. [5]

Answer : (a) Let a spherical surface separate a rarer medium of refractive index  $n_1$  from second medium of refractive index  $n_2$ . Let C be the centre of curvature and  $R = MC$  be the radius of the surface.



Consider a point object O lying on the principal axis of the surface. Let a ray starting from O incident normally on the surface along OM and pass straight. Let another ray of light incident on NM along ON and refract along NI.

From M, draw MN perpendicular to OI.

The above figure shows the geometry of formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R.

Let us make the following assumptions :

- The aperture of the surface is small as compared to the other distance involved.
- The object consists only of a point lying on the principal axis of the spherical refracting surface.

$$\tan \angle NOM = \frac{MN}{OM}$$

$$\tan \angle NCM = \frac{MN}{MI}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

For  $\Delta NOC$ ,  $i$  is the exterior angle.

$$\therefore i = \angle NOM + \angle NCM$$

For small angles  $\tan i = i$ ,

$$i = \frac{MN}{OM} + \frac{MN}{MC} \quad \dots(i)$$

Similarly,  $r = \angle NCM - \angle NIM$

$$r = \frac{MN}{MC} = \frac{MN}{MI} \quad \dots(ii)$$

By Snell's law,

$$n_1 \sin i = n_2 \sin r$$

For small angles,

$$n_1 i = n_2 r$$

Substituting the values of  $i$  and  $r$  from (i) and (ii), we obtain

$$n_1 \left( \frac{MN}{OM} + \frac{MN}{MC} \right) = n_2 \left( \frac{MN}{MC} + \frac{MN}{MI} \right)$$

or, 
$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad \dots(iii)$$

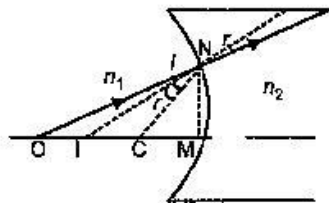
Applying new Cartesian sign conventions, we get

$$OM = -u, MI = +v, MC = +R$$

Substituting these values in equation (iii), we obtain

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \dots(iv)$$

(b)



Now the image  $I$  acts as virtual object for the second surface that will form real image at  $I$ . As refraction takes place from denser to rarer medium

$$\therefore -\frac{n_2}{v} + \frac{n_1}{v'} = \frac{n_1 - n_2}{R} \quad \dots(v)$$

Adding (iv) and (v) we get  $\left[ \because k = \frac{1}{4\pi\epsilon_0} \right]$

$$\frac{n_1}{v'} - \frac{n_1}{u} = (n_2 - n_1) \left[ \frac{1}{R} - \frac{1}{R'} \right]$$

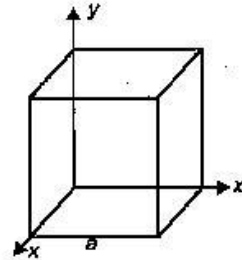
$$\frac{1}{f} = (n_{21} - 1) \left[ \frac{1}{R} - \frac{1}{R'} \right]$$

$$\left[ \because n_{21} = \frac{n_2}{n_1}, \frac{1}{f} = \frac{1}{v'} - \frac{1}{u} \right]$$

26. (a) An electric dipole of dipole moment  $\vec{p}$  consists of point charges  $+q$  and  $-q$  separated by a distance  $2a$  apart. Deduce the expression for the electric field  $\vec{E}$  due to the dipole at a distance  $x$  from the centre of the dipole on its axial line in terms of the dipole moment  $\vec{p}$ . Hence show that in the limit

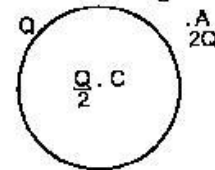
$$x \gg a, \vec{E} \rightarrow \frac{2\vec{p}}{4\pi\epsilon_0 x^3}$$

- (b) Given the electric field in the region  $\vec{E} = 2x\hat{i}$ , find the net electric flux through the cube and the charge enclosed by it. [5]



OR

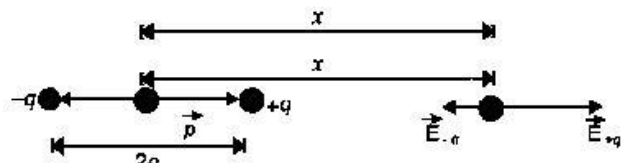
- (a) Explain, using suitable diagrams, the difference in the behaviour of a (i) conductor and (ii) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.
- (b) A thin metallic spherical shell of radius carries a charge  $Q$  on its surface. A point charge  $\frac{Q}{2}$  is placed at its centre  $C$  and another charge  $+2Q$  is placed outside the shell at a distance  $x$  from the centre as shown in the figure. Find (i) the force on the charge at the centre of shell and at the point  $A$ , (ii) the electric flux through the shell.



Answer : (a) Electric field at a point on the axial line

$$|\vec{E}_{-q}| = \frac{kq}{(x-a)^2}$$

$$|\vec{E}_{+q}| = \frac{kq}{(x+a)^2}$$



$$\vec{E} = \vec{E}_{+q} - \vec{E}_{-q} = \frac{kq}{(x-a)^2} - \frac{kq}{(x+a)^2}$$

$$\Rightarrow |\vec{E}| = \frac{kq4ax}{(x^2 - a^2)^2}$$

$$\vec{E} = \frac{2k\vec{p}x}{(x^2 - a^2)^2} \text{ (Parallel to } \vec{p}) \quad \left[ \because \vec{p} = 2aq \right]$$

If  $x \gg a$ ,  $\vec{E} = \frac{2\vec{p}}{4\pi\epsilon_0 x^3}$ .

In vector form,  $\vec{E} = \frac{2\vec{p}}{4\pi\epsilon_0 x^3}$   $\left[ \because k = \frac{1}{4\pi\epsilon_0} \right]$

(b) Since, the electric field is parallel to the faces and parallel to  $xy$  and  $xz$  planes, the electric flux through them is zero.

Electric flux through the left face

$$\begin{aligned} \phi_L &= (E_L) (a^2) \cos 180^\circ \\ &= (0) (a^2) \cos 180^\circ = 0 \end{aligned}$$

Electric flux through the right face,

$$\begin{aligned} \phi_R &= (E_R) (a^2) \cos 0^\circ \\ &= (2a) (a^2) \times 1 \\ &= 2a^3 \end{aligned}$$

$$\text{Total flux } (\phi) = 2a^3 = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\therefore q_{\text{enclosed}} = 2a^3 \epsilon_0$$

OR

(a) (i) Conductor :

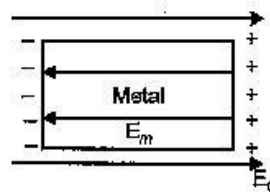
Let  $E_0$  = external field, and

$E_m$  = internal field created by the redistribution of electrons inside the metal.

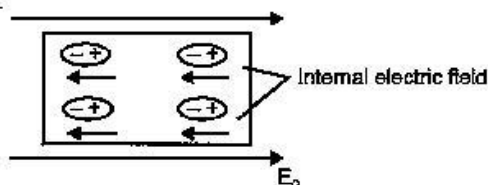
When a conductor like a metal is subjected to external electric field, the electrons experience a force in the opposite direction collecting on the left hand side.

A positive charge is therefore induced on the right hand side. This creates an opposite electric field ( $E_m$ ) that balances out ( $E_0$ ).

$\therefore$  The net electric field inside the conductor becomes zero.



(ii) Dielectric :



When an external electric field is applied, dipoles are created (in case of non-polar dielectrics). The placement of dipoles is as shown in the given figure. An internal electric field is created which reduces the external electric field.

Polarization of dielectric ( $P$ ) is defined as the dipole moment per unit volume of the polarized dielectric.

$$P = \chi_e \epsilon_0 E$$

Where  $\chi_e$  = Susceptibility

$E$  = Electric field

(b) (i) Net force on the charge  $\frac{Q}{2}$ , placed at the centre of the shell is zero.

Net Force on charge  $2Q$  kept at a point A

$$F = E \times 2Q = \frac{1 \cdot \left(\frac{3Q}{2}\right) 2Q}{4\pi\epsilon_0 r^2} = \frac{k3Q^2}{r^2}$$

(ii) Electric flux through the shell,

$$\phi = \frac{Q}{2\epsilon_0} \quad \left[ \because \phi = \frac{q_{\text{enclosed}}}{\epsilon_0} \right]$$

Note: All questions of Delhi Set II are from Delhi Set I and Delhi Set III are from Delhi Set II.

## Physics 2014 (Outside Delhi)

## SET I

Time allowed : 3 hours

Maximum marks : 70

1. Using the concept of force between two infinitely long parallel current carrying conductors, define one ampere of current. [1]

**Answer :** One ampere is the current which when flowing through each of the two parallel uniform long linear conductors placed in free space at a distance of one metre from each other will attract or repel each other with a force of  $2 \times 10^{-7}$  N per metre of their length.

2. To which part of the electromagnetic spectrum does a wave of frequency  $5 \times 10^{19}$  Hz belong ? [1]

**Answer :** The frequency  $5 \times 10^{19}$  Hz lies in the gamma region of the electromagnetic spectrum.

3. What is the force between two small charges of  $2 \times 10^{-7}$  C and  $3 \times 10^{-7}$  C placed 30 cm apart in air ? [1]

